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Green-Schwarz superstring action in a curved magnetic Ramond-Ramond background

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Abstract: We derive the complete covariant action for the type IIA superstring in a simple $D = 10$ background which represents a 7-brane with a magnetic Ramond-Ramond vector field (and is U-dual to the Kaluza-Klein Melvin solution). This curved background can be obtained by dimensional reduction from a flat (but topologically non-trivial) $D = 11$ space-time. The action of a supermembrane propagating in this flat $D = 11$ space is straightforward to write down. The explicit form of the superstring action is then obtained by double dimensional reduction of the supermembrane action. In the light-cone gauge the action contains only quadratic and quartic terms in fermions.

Keywords: p-branes, M-Theory, Superstrings and Heterotic Strings.

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1. Introduction

One interesting problem in superstring theory is to understand how fundamental strings propagate in the presence of background fields corresponding to the Ramond-Ramond (RR) sector of type II superstring spectrum. This is important, in particular, in view of the presence of D-branes [1] in the theory. It is natural to try to address this question using the Green-Schwarz (GS) formulation [2] in which the string action in a RR background is local and supersymmetric. The leading-order terms in the coupling of GS superstring to RR fields can be constructed [3] in the light-cone gauge using the known light-cone gauge GS vertex operators [4]. The formal superspace expressions for the κ-invariant type II GS superstring actions in generic on-shell $N = 2, D = 10$ supergravity backgrounds were found in [5, 6] (and studied in [7]), but the explicit component form of the actions was not worked out. A different approach to the construction of manifestly $N = 2, D = 4$ supersymmetric world sheet σ-model for compactified $D = 4$ Type II superstring was presented in [8]. The dilaton dependence of RR coupling terms in GS action was explicitly demonstrated in [9]. Certain leading component terms in $D = 11$ membrane action [10] and thus [6] in $D = 10$ string action were recently determined in [11].

Given that finding the complete expression for the covariant superstring action in terms of the component fields (x, θ) to all orders in θ's in a generic supergravity background is obviously a complicated problem,[1] one may first try to solve it for some special cases of the RR backgrounds. In this paper we shall consider a simple example of the RR background that solves the type IIA supergravity equations of motion. It

[1]GS action is complicated even in curved NS-NS backgrounds, but in the absence of RR fields one may use the well-understood NS sigma-model representation for the superstring action.

represents a (non-supersymmetric) magnetic 7-brane of type IIA theory which is the RR analogue (actually, U-dual) of the NS-NS Kaluza-Klein Melvin ('flux tube') background. The latter was previously discussed at the field theory level [12, 13, 14, 15] and at the string theory level [16] (the corresponding type II superstring model is exactly soluble so that its mass spectrum and partition function can be explicitly determined [16]).

This background can be obtained by dimensional reduction from a simple eleven-dimensional space-time which is *flat* (and thus should be an exact solution of M-theory) but topologically non-trivial. Its non-trivial 3-dimensional part is obtained by factorizing $(R^2)_{r,\varphi} \times (S^1)_y$ over the group generated by translations in two angular directions.[2] If the 11-th direction is different from y, this $D = 11$ background reduces to a similar flat one in $D = 10$ which describes the NS-NS Kaluza-Klein Melvin solution upon further reduction to $D = 9$ (as in [16] in what follows we shall refer to this flat $D = 10$ background as Kaluza-Klein Melvin model). If instead we reduce along $y = x_{11}$ we end up with a curved type IIA $D = 10$ background with a non-trivial RR vector potential representing a magnetic flux 'tube' (7-brane).

The key technical point is that the GS string action in this $D = 10$ RR background can be found by the double dimensional reduction [6] from the explicitly known supermembrane action [10] in the corresponding *flat* $D = 11$ space (the relevant membrane action is related to the standard action written in terms of flat cartesian coordinates by a simple coordinate transformation).[3] The resulting covariant string action is non-trivial, containing higher powers of θ's. After fixing the light-cone gauge, the action simplifies to a form containing only quadratic and quartic terms in fermions.

2. Magnetic Ramond-Ramond background

The ten-dimensional background that we shall consider is a solution type IIA supergravity with the relevant part of the action being

$$S = \int d^{10}x\sqrt{-G}\Big(e^{-2\phi}[R + 4(\partial\phi)^2] - \tfrac{1}{2}F_{mn}F^{mn}\Big) \ . \tag{2.1}$$

It is given by (q =const)

$$ds^2_{10} = f(r)[\,-dt^2 + dx_1^2 + ... + dx_7^2 + dr^2 + r^2 f^{-2}(r)d\varphi^2] \ , \tag{2.2}$$

$$e^{2\phi} = f^3(r) \ , \qquad A = qr^2 f^{-2}(r)d\varphi \ , \qquad f \equiv (1 + q^2r^2)^{1/2} \ , \tag{2.3}$$

[2] In the coordinates where $ds^2 = dr^2 + r^2d\theta^2 + dy^2$ ($\theta = \varphi + qy$) one identifies the points $(r, \theta, y) = (r, \theta + 2\pi n + 2\pi qRm, y + 2\pi Rm)$ (n, m =integers), i.e. combines the shift by $2\pi R$ in y with a rotation by an arbitrary angle $2\pi qR$ in the 2-plane. The fixed r section is a 2-torus (with r-dependent conformal factor and complex modulus) which degenerates into a circle at $r = 0$. The space is actually regular everywhere, including $r = 0$, as becomes clear in rewriting the metric in cartesian coordinates.

[3] The general superspace expression for a GS superstring action in an arbitrary type IIA supergravity background was originally derived [6] by starting from the membrane action [10] in a curved $D = 11$ supergravity background.

where A is the RR 1-form. It can be obtained by dimensional reduction from the $D = 11$ background with trivial 3-form field $A_{\mu\nu\rho} = 0$ and the metric

$$ds_{11}^2 = -dt^2 + dx_1^2 + ... + dx_7^2 + dr^2 + r^2(d\varphi + qdy)^2 + dy^2 \ . \tag{2.4}$$

Here $\varphi \equiv \varphi + 2\pi$ and $y \equiv x_{11}$ has period $2\pi R_{11}$ so that this metric is topologically non-trivial if $qR_{11} \neq n$. Since the metric is locally flat this an exact solution to the equations of motion of eleven-dimensional supergravity *and* M-theory (all possible higher-order curvature corrections vanish). If the dimensional reduction is done along one of the coordinates $x_1, ..., x_7$ one obtains the $D = 10$ metric representing the NS-NS Kaluza-Klein Melvin background discussed in [12, 13, 14, 15]. Here instead we shall consider the dimensional reduction along y. By writing the metric (2.4) as

$$ds_{11}^2 = -dt^2 + dx_1^2 + ... + dx_7^2 + dr^2 + r^2 f^{-2}(r) d\varphi^2 + f^2(r)[dy + qr^2 f^{-2}(r) d\varphi]^2 \ , \tag{2.5}$$

we obtain the above type IIA solution (2.2) with the magnetic RR vector field. This RR background is obviously related to the NS-NS Melvin solution by U-duality.[4]

The background (2.2) is a curved space-time corresponding to an axially symmetric magnetic RR 'flux tube' (7-brane, see below). If the string coupling $e^{\phi_0 + \phi(r)}$ is chosen to be small at $qr \ll 1$, it becomes large at $qr \gg 1$, so that the geometry is ten-dimensional close to the core of 7-brane (inside the flux tube) and becomes eleven-dimensional far from it.

It is useful to make the following coordinate transformation:

$$f(r)\frac{dr}{r} = \frac{d\rho}{\rho} \ , \qquad \rho = 2q^{-1} e^{f(r)-1} [\frac{f(r) - 1}{f(r) + 1}]^{1/2} \ , \tag{2.6}$$

putting the solution into the explicit rotationally symmetric 7-brane form

$$ds_{10}^2 = H_1(\rho)(- dt^2 + dx_1^2 + ... + dx_7^2) + H_2(\rho)(d\rho^2 + \rho^2 d\varphi^2) \ , \tag{2.7}$$

$$e^{2\phi} = H_1^3(\rho) \ , \qquad A = \frac{q\rho^2 H_2(\rho)}{H_1(\rho)} d\varphi \ , \qquad H_1 \equiv f[r(\rho)] \ , \quad H_2 \equiv \frac{r^2(\rho)}{\rho^2 f[r(\rho)]} \ .$$

Like the U-dual KK Melvin background, this 7-brane solution breaks all supersymmetries.[5] The behavior at small and large ρ is as follows:

$$r(\rho) \cong \rho \ (1 - \tfrac{1}{4}q^2\rho^2) \ , \quad H_1(\rho) \cong 1 + \tfrac{1}{2}q^2\rho^2 \ , \quad H_2(\rho) \cong 1 - q^2\rho^2 \qquad \text{if } q\rho \ll 1$$

[4]Explicitly, the Kaluza-Klein Melvin model in type IIA theory is obtained from the RR Melvin model by applying T-duality along x_7, then S-duality in IIB theory, and then T-duality along x_7 to get back to IIA theory. Though the IIA KK Melvin model described by flat $D = 10$ space is an exact solution of string theory, this may not apply to the RR Melvin model since due to lack of supersymmetry there may be both perturbative and non-perturbative corrections to the duality transformations. Let us note also that KK Melvin solution of type IIB theory is related to that of type IIA theory by a trivial T-duality transformation along one of the flat 'spectator' coordinates.

[5]The $D = 11$ Killing spinor ε does not satisfy the periodic boundary condition in y (unless $qR_{11} = 2n$) since $\varepsilon(x, y + 2\pi R_{11}) = \exp[\frac{1}{2}qR_{11}\Gamma_8\Gamma_9] \ \varepsilon(x, y)$, where $8, 9$ correspond to the directions in the (r, φ) plane.

$$q\ r(\rho) \cong \log(q\rho)\ , \quad H_1(\rho) \cong \log(q\rho)\ , \quad H_2(\rho) \cong \frac{\log(q\rho)}{q^2\rho^2} \qquad \text{if } q\rho \gg 1\ .$$

The gauge field strength dA is constant near the origin and asymptotically approaches zero at large ρ.

Our aim below will be to find the exact expression for the action of the GS string propagating in the background (2.2),(2.3). We shall obtain it by double dimensional reduction along $x_{11} = y$ from the $D = 11$ action of a membrane moving in the metric (2.4).

The reduction of the same membrane action along the 'trivial' direction x_7 leads to the action for GS string in the Kaluza-Klein Melvin background. In the light-cone gauge, the non-trivial part of the latter action (without terms corresponding to free bosonic directions $x_1, ..., x_6$) has the form [16][6]

$$L_{\text{KK Melvin}} = \partial_+ y \partial_- y + D_+ x_s D_- x_s + iS_R \mathcal{D}_+ S_R + iS_L \mathcal{D}_- S_L\ , \tag{2.8}$$

$$(D_a)_{st} \equiv \delta_{st}\partial_a - q\epsilon_{st}\partial_a y\ , \qquad \mathcal{D}_i \equiv \partial_a - \tfrac{1}{4}q\epsilon_{st}\Gamma_{st}\partial_a y\ , \qquad s,t = 8,9\ . \tag{2.9}$$

3. Green-Schwarz action

The action for the $D = 11$ supermembrane coupled to a $D = 11$ supergravity background is given by [10]

$$I = -\tfrac{1}{2}T_3 \int d^3\xi \Big[\sqrt{-g}(g^{ij}\hat\Pi_i^{\hat\mu}\hat\Pi_j^{\hat\nu}\eta_{\hat\mu\hat\nu} - 1) - \tfrac{1}{3}\epsilon^{ijk}\hat\Pi_i^A\hat\Pi_j^B\hat\Pi_k^C A_{CBA}\Big]\ . \tag{3.1}$$

Here g_{ij} is the auxiliary 3d metric ($i = 1,2,3$), $\hat\Pi_i^A = \partial_i Z^M \hat E_M^A(Z)$, $Z^M = (x^\mu, \theta^\alpha)$ ($\mu = 0,1,...,9,11$, $\alpha = 1,...,32$), $A_{CBA} = A_{CBA}(Z)$ ($A = (\hat\mu, \hat\alpha)$ are superspace tangent indices).[7] As was shown in [10, 6], this action is invariant under κ-symmetry provided the background satisfies the superspace equations of on-shell $D = 11$ supergravity [18, 19].

In flat superspace the action (3.1) takes the following explicit form [10]

$$\begin{aligned} I = -\tfrac{1}{2}T_3 \int d^3\xi \Big[&\sqrt{-g}g^{ij}(\partial_i x^\mu - i\bar\theta\Gamma^\mu\partial_i\theta)(\partial_j x^\mu - i\bar\theta\Gamma^\mu\partial_j\theta) - \sqrt{-g} \\ &+ i\epsilon^{ijk}\bar\theta\Gamma_{\mu\nu}\partial_i\theta(\partial_j x^\mu\partial_k x^\nu - i\partial_j x^\mu\bar\theta\Gamma^\nu\partial_k\theta - \tfrac{1}{3}\bar\theta\Gamma^\mu\partial_j\theta\bar\theta\Gamma^\nu\partial_k\theta)\Big] \equiv I_N + I_{WZ}\ . \end{aligned} \tag{3.2}$$

[6]It is useful to recall also that under T-duality along y the KK Melvin background is transformed into a curved background described by the string model with the following bosonic part (this is a special case of a 3-parameter class of magnetic models solved in [17]): $\tilde L = \partial_+ r\partial_- r + F(r)r^2(\partial_+\varphi + q\partial_+\tilde y)(\partial_-\varphi - q\partial_-\tilde y) + \partial_+\tilde y\partial_-\tilde y + \mathcal{R}(\phi_0 + \frac{1}{2}\ln F)$, $F \equiv (1+q^2r^2)^{-1}$, $\mathcal{R} \equiv \frac{1}{4}\alpha'\sqrt{g}R^{(2)}$. This model is equivalent to the KK Melvin model at the CFT level, i.e. it has, in particular, the same mass spectrum [16].

[7]We shall assume that Γ^μ ($\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = \text{diag}(-1,1...,1)$) are chosen to be real and the Majorana spinors θ are real, $\bar\theta = \theta^T C$, $C = \Gamma^0$. Note that $C_{\alpha\beta}, (C\Gamma^{\mu_1\mu_2\mu_3})_{\alpha\beta}, (C\Gamma^{\mu_1\mu_2\mu_3\mu_4})_{\alpha\beta}$ are antisymmetric while $(C\Gamma^\mu)_{\alpha\beta}, (C\Gamma^{\mu_1\mu_2})_{\alpha\beta}, (C\Gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5})_{\alpha\beta}$ are symmetric in α, β. In particular, $\bar\theta\Gamma^\mu\theta = 0$.

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Eliminating the auxiliary metric, the first 'even' term of this action can be written in the standard form

$$I_N = -T_3 \int d^3\xi \sqrt{-\mathrm{det}_3 h_{ij}} \ , \qquad h_{ij} = (\partial_i x^\mu - i\bar\theta\Gamma^\mu\partial_i\theta)(\partial_j x^\mu - i\bar\theta\Gamma^\mu\partial_j\theta) \ . \tag{3.3}$$

Let us now consider the background (2.4). The metric (2.4) is flat since locally it can be obtained from the standard cartesian metric of $R^{1,7} \times R^2 \times S^1$ by a y-dependent rotation in the (x_8, x_9) plane. Therefore, the corresponding supermembrane action is formally related to the flat space membrane action (3.2) by the substitution

$$\theta \to \theta_{\rm flat} = e^{-\frac{1}{2}q\Gamma_* y}\ \theta \ , \qquad \Gamma_* \equiv \Gamma_8\Gamma_9 \ ,$$

$$x^8 + ix^9 = re^{i\varphi} \ \to \ x^8_{\rm flat} + ix^9_{\rm flat} = e^{iqy}(x^8+ix^9) \ , \qquad x^\mu_{\rm flat} = x^\mu \ , \quad \mu \neq 8,9 \ , \tag{3.4}$$

where (x^μ, θ^α) are the 'true' superspace coordinates with correct periodicity (the 'flat' coordinates are not single-valued in the y direction). The action (3.2) then becomes

$$I = -T_3 \int d^3\xi \sqrt{-\mathrm{det}_3 h_{ij}} + I_{WZ} \ , \tag{3.5}$$

$$h_{ij} = (D_i x^\mu - i\bar\theta\Gamma^\mu\mathcal{D}_i\theta)(D_j x^\mu - i\bar\theta\Gamma^\mu\mathcal{D}_j\theta) \ , \tag{3.6}$$

$$I_{WZ} = -\tfrac{1}{2}iT_3 \int d^3\xi \ \epsilon^{ijk}\bar\theta\Gamma_{\mu\nu}\mathcal{D}_i\theta(D_j x^\mu D_k x^\nu - iD_j x^\mu\bar\theta\Gamma^\nu\mathcal{D}_k\theta - \tfrac{1}{3}\bar\theta\Gamma^\mu\mathcal{D}_j\theta\bar\theta\Gamma^\nu\mathcal{D}_k\theta) \ , \tag{3.7}$$

where the covariant derivatives are defined as follows (they are the same as in (2.9))

$$D_i x_s \equiv \partial_i x_s - q\epsilon_{st}x_t\partial_i y \ , \qquad D_i x_u \equiv \partial_i x_u \ , \qquad D_i x_{11} = D_i y \equiv \partial_i y \ ,$$

$$\mathcal{D}_i\theta \equiv (\partial_i - \tfrac{1}{2}q\Gamma_*\partial_i y)\theta \ , \tag{3.8}$$

where $s,t = 8,9$ and $u = 0,1,..,7$.

In order to obtain the type IIA GS superstring action in $N=2, D=10$ supergravity background (2.2),(2.3) we perform the double dimensional reduction [6] by splitting the world-volume and space-time coordinates as $\xi^i \to (\xi^a, \xi^3)$, $x^\mu \to (x^m, x^{11} \equiv y)$, assuming that $\partial_3 g_{ij} = 0$, $\partial_3 x^m = 0$, $\partial_3\theta = 0$, and setting $y = R_{11}\xi^3$. Here and in what follows $a,b = 1,2,\ \ m,n = 0,1,...,9$ (repeated indices are summed with Minkowski metric).

The WZ term (3.7) then reduces to ($T_2 = 2\pi R_{11} T_3$)

$$I_{WZ} = I^{(0)}_{WZ} + I^{(q)}_{WZ} \ , \tag{3.9}$$

$$I^{(0)}_{WZ} = -\tfrac{1}{2}iT_2 \int d^2\xi \ \epsilon^{ab}\bar\theta\Gamma_m\Gamma_{11}\partial_a\theta(2\partial_b x^m - i\bar\theta\Gamma^m\partial_b\theta) \ , \tag{3.10}$$

$$\begin{aligned} I^{(q)}_{WZ} = -\tfrac{1}{2}iqT_2 \int d^2\xi \ \epsilon^{ab} \Big[& \epsilon_{ts}x_s\bar\theta\Gamma_{tm}\partial_a\theta \ (2\partial_b x^m - i\bar\theta\Gamma^m\partial_b\theta) \\ & -i\epsilon_{ts}x_s\bar\theta\Gamma_t\Gamma_{11}\partial_a\theta \ \bar\theta\Gamma_{11}\partial_b\theta - \tfrac{1}{2}\bar\theta\Gamma_{mn}\Gamma_*\theta(\partial_a x^m\partial_b x^n - i\partial_a x^m\bar\theta\Gamma^n\partial_b\theta \\ & -\tfrac{1}{3}\bar\theta\Gamma^m\partial_a\theta\bar\theta\Gamma^n\partial_b\theta) - \tfrac{1}{2}\bar\theta\Gamma^n\Gamma_*\theta \ \bar\theta\Gamma_{mn}\partial_b\theta(i\partial_a x^m - \tfrac{2}{3}\bar\theta\Gamma^m\partial_a\theta) \\ & + \tfrac{1}{2}\bar\theta\Gamma_m\Gamma_{11}\Gamma_*\theta(i\partial_a x^m\bar\theta\Gamma^{11}\partial_b\theta + \tfrac{2}{3}\bar\theta\Gamma^m\partial_a\theta \ \bar\theta\Gamma^{11}\partial_b\theta) \\ & - \tfrac{1}{2}\bar\theta\Gamma_m\Gamma_{11}\partial_b\theta(i\partial_a x^m\bar\theta\Gamma^{11}\Gamma_*\theta \ - \tfrac{2}{3}\bar\theta\Gamma^{11}\Gamma_*\theta \ \bar\theta\Gamma^m\partial_a\theta + \tfrac{2}{3}\bar\theta\Gamma^m\Gamma_*\theta \ \bar\theta\Gamma^{11}\partial_a\theta)\Big] \end{aligned} \tag{3.11}$$

Let us now consider the more complicated induced metric term I_N (the first term in (3.5)). In general,

$$\det{}_3 h_{ij} = \det{}_2 f_{ab} \ , \qquad f_{ab} = h_{33}^{1/2}(h_{ab} - h_{33}^{-1} h_{a3} h_{b3}) \ , \tag{3.12}$$

where

$$\begin{aligned} h_{ab} &= (\partial_a x^m - i\bar\theta\Gamma^m\partial_a\theta)(\partial_b x^m - i\bar\theta\Gamma^m\partial_b\theta) - \bar\theta\Gamma_{11}\partial_a\theta\bar\theta\Gamma_{11}\partial_b\theta \ , \\ h_{a3} &= R_{11}\Big(- i\bar\theta\Gamma_{11}\partial_a\theta + q[\epsilon_{ts}x^t(\partial_a x^s - i\bar\theta\Gamma^s\partial_a\theta) \\ &\quad + \tfrac{1}{2}i(\partial_a x^u - i\bar\theta\Gamma^u\partial_a\theta)\bar\theta\Gamma^u\Gamma_*\theta + \tfrac{1}{2}\bar\theta\Gamma_{11}\partial_a\theta\bar\theta\Gamma_{11}\Gamma_*\theta\ ]\Big) \equiv R_{11}\bar h_{a3} \ , \\ h_{33} &= (R_{11})^2\Big[(1+\tfrac{1}{2}iq\bar\theta\Gamma_{11}\Gamma_*\theta)^2 + q^2x_sx_s - \tfrac{1}{4}q^2(\bar\theta\Gamma^u\Gamma_*\theta)^2\Big] \equiv (R_{11})^2\bar h_{33} \ , \end{aligned} \tag{3.13}$$

where we have used that $\bar\theta\Gamma^\mu\theta = 0$. The corresponding part of the string action $\int d^2\xi\sqrt{-\det_2 f_{ab}}$ can be represented in the usual form with an auxiliary 2d metric g_{ab}: $\frac{1}{2}\int d^2\xi\sqrt{-g}g^{ab}f_{ab}$.

Choosing the standard conformal gauge $\sqrt{-g}g^{ab} = \eta^{ab}$ we finish with the following GS action: $I = I_N + I_{WZ}$, where I_{WZ} was given in (3.10),(3.11), and

$$I_N = -\tfrac{1}{2}T_2\int d^2\xi\ \bar h_{33}^{1/2}\ (h_{aa} - \bar h_{33}^{-1}\bar h_{a3}\bar h_{a3}) \ , \tag{3.14}$$

where the repeated indices are contracted with the flat 2d metric. It is easy to check that the bosonic terms in this action have the standard σ-model form $G_{mn}(x)\partial_a x^m\partial_a x^n$ corresponding to the metric (2.2). Indeed, note that $x_8 = r\cos\varphi,\ x_9 = r\sin\varphi$, and

$$\bar h_{33} = f^2(r) + O(\theta^2) \ , \qquad dx_sdx_s - f^{-2}(r)(\epsilon_{st}x_sdx_t)^2(r)r^4d\varphi^2 = dr^2 + f^{-2}(r)r^2d\varphi^2 \ .$$

In the $q=0$ limit the action reduces to the standard expression for the GS action in the flat $D=10$ space [2]

$$\begin{aligned} I(q=0) = -\tfrac{1}{2}T_2\int d^2\xi\ \Big[&(\partial_a x^m - i\bar\theta\Gamma^m\partial_a\theta)(\partial_a x^m - i\bar\theta\Gamma^m\partial_a\theta) \\ &+\ i\epsilon^{ab}\bar\theta\Gamma_m\Gamma_{11}\partial_a\theta(2\partial_b x^m - i\bar\theta\Gamma^m\partial_b\theta)\Big] \ , \end{aligned} \tag{3.15}$$

written here in the $D=11$ notation for the Majorana spinors.[8] For $q \neq 0$ the action has a complicated form, especially because of the θ-dependent $h_{33}^{1/2}$ factor in (3.14) (which is equal to $e^{\frac{2}{3}\Phi(x,\theta)}$ where Φ is the $D=10$ dilaton superfield).[9] The presence of higher-order fermionic terms in the action reflects the curvature of the background. The

[8]Relation to the two $D=10$ Majorana-Weyl spinors is $\theta = (\theta^1,\theta^2),\ \Gamma_{11} = \mathrm{diag}(I,-I),\ \Gamma^m = \begin{pmatrix} 0 & \gamma^m \\ \gamma^m & 0 \end{pmatrix}$.

[9]The explicit expressions for the $D=10$ superfields representing the supergeometry of our background can be read off from the superstring action by comparing it to the general result [6] of the double dimensional reduction of the membrane action in curved $D=11$ space.

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WZ term contains terms which are of first order only in the magnetic flux parameter q which describe, in particular, the coupling to the RR vector field strength.[10] Since the action explicitly depends on θ (and not only on $\partial_a\theta$) it does not have obvious global translational fermionic symmetries (indeed, the magnetic background breaks all supersymmetries).

4. Light-Cone gauge

The action simplifies dramatically once one fixes the light-cone gauge. Since the action is κ-symmetric we are free to impose the condition

$$\Gamma^+\theta = 0 \ , \qquad \Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^1) \ . \tag{4.1}$$

Note that our background is non-trivial only in the 8, 9 directions, i.e. the choice of the light-cone gauge 'commutes' with the rotation (3.4) (in particular, $[\Gamma^+, \Gamma_\mu] = 0$ for $\mu = 8, 9, 11$). Instead of using (4.1) directly in the string action which follows from (3.5) we may obtain the same final expression in a more straightforward way by first choosing the fermionic light-cone gauge in the free membrane action (3.2) (assuming $\partial_3\theta = 0$) and *then* applying the rotation (3.4) and dimensional reduction. The standard observation is that if $[\mathcal{O}, \Gamma^+] = 0$ and θ satisfies (4.1) then $\bar\theta\Gamma^\mu\mathcal{O}\theta$ is equal to zero unless the index μ takes the value $-$.[11] This implies, in particular, that all quartic fermionic terms in h_{ab}, h_{a3} and all fermionic terms in h_{33} in (3.13) vanish, so that $\bar h_{33}$ becomes θ-independent and equal to $f^2 = 1 + q^2 x_s x_s$. Using (4.1) in the flat membrane action (3.2) and then making the rotation (i.e. replacing the derivatives by the covariant derivatives (3.8)) we find the action which contains only terms which are at most bilinear in fermions,

$$\begin{aligned} I = -\tfrac{1}{2}T_3 \int d^3\xi \Big[&\sqrt{-g}g^{ij}(D_i x^\mu D_j x^\mu - 2i\bar\theta\Gamma^- \mathcal{D}_i\theta\partial_j x^+) - \sqrt{-g} \\ &+ 2i\epsilon^{ijk}\bar\theta\Gamma^p\Gamma^-\mathcal{D}_i\theta D_j x^p \partial_k x^+ \Big] \ , \end{aligned} \tag{4.2}$$

where $p = 2, ..., 9, 11$. The term with $p = 11$ in the WZ part of (4.2) leads to the standard WZ term $2i\epsilon^{ab}\bar\theta\Gamma^-\Gamma_{11}\partial_a\theta\partial_b x^+$ in the light-cone gauge GS string action.

The elimination of the auxiliary metric after dimensional reduction reintroduces a quartic fermionic term. Equivalently, a θ^2 term in h_{a3} in (3.13) gives rise to the θ^4 term in (3.14). As follows from (3.13),(4.1)

$$h_{ab} = \partial_a x^m \partial_b x^m - 2i\partial_a x^+ \bar\theta\Gamma^-\partial_a\theta \ , \quad \bar h_{a3} = q(\epsilon_{ts}x_t\partial_a x_s + \tfrac{1}{2}i\partial_a x^+ \bar\theta\Gamma^-\Gamma_*\theta) \ ,$$

[10] In particular, (3.11) contains the term $e^\phi F_{st}\epsilon_{ab}\partial_a x^m \partial_b x^n \bar\theta\Gamma_{mn}\Gamma_{st}\theta$ (where F_{st} is the RR vector field strength) expected on the general grounds (e.g., from comparison with the RR vertex operator). For $m, n = 8, 9$ this is the term derived from $D = 11$ membrane action in [9].

[11] As usual, for Majorana spinors $\bar\theta = \theta^T C$, $C\Gamma_m = -\Gamma_m^T C$, so that $\Gamma^+\theta = 0$ implies $\bar\theta\Gamma^+ = 0$. One is also to note that $1 = \Gamma^+\Gamma^- + \Gamma^-\Gamma^+$ may be inserted in the fermionic bilinears.

and $\bar{h}_{33} = 1 + q^2 x_s x_s = f^2(r)$. The light-cone string model we get is thus the following

$$I = -\tfrac{1}{2}T_2 \int d^2\xi \Big[f(r)(\partial_a x^m \partial_a x^m - 2i\partial_a x^+ \bar\theta \Gamma^- \partial_a \theta) \\ - q^2 f^{-1}(r)(\epsilon_{ts} x_t \partial_a x_s + \tfrac{1}{2} i \partial_a x^+ \bar\theta \Gamma^- \Gamma_* \theta)^2 \\ + 2i\epsilon^{ab}\partial_b x^+ [\bar\theta \Gamma^- \Gamma_{11} \partial_a \theta - q(\epsilon_{st} x_t \bar\theta \Gamma^- \Gamma_s \partial_a \theta + \tfrac{1}{2}\partial_a x^p \bar\theta \Gamma^- \Gamma_* \Gamma^p \theta)] \Big] . \qquad (4.3)$$

The θ^4 term reflects the curvature of the background, while the RR coupling term is only quadratic in fermions.[12]

Next, one may fix the remaining conformal invariance by the bosonic light-cone gauge $x^+ = c + p^+ \xi^1$ (the 7-brane background (2.2),(2.3)) has $SO(1,7)$ Lorentz symmetry). This gauge choice could actually be made already at the level of the membrane action (4.2) (as in the standard flat space case [10]) since it 'commutes' with the rotation in the $(8,9)$ plane. In contrast to the flat KK Melvin model case (2.8), however, the action (4.3) does not take a manifestly 2d Lorentz-covariant form with θ replaced by a pair of 2d spinors (the 2d parity even and odd terms in (4.3) do not combine as they do in the case of a flat target space since the even term here has the extra factor of $f(r)$). Nevertheless, since the action is at most quartic in fermions, it may be useful to perform a triality transformation on spinors to convert them into $SO(8)$ vectors, and that may lead to a more convenient framework to demonstrate the conformal invariance of this model at the quantum level.

The flat space membrane action describes a non-trivial interacting model even after fixing the light-cone gauge [10, 20]. While the double dimensional reduction of it along one of the 'spectator' x^m, $m = 1, ..., 7$ coordinates leads to a simple quadratic action (2.8) of KK Melvin model (which can be solved in terms of free oscillators by 'undoing' the rotation in the plane [16]), the reduction along y leads to the non-trivial interacting theory (4.3) , which can no longer be put into a flat or quadratic form by any obvious redefinition of the supercoordinates.

5. Discussion

Similar considerations based on double dimensional reduction of the $D = 11$ membrane action allow one to find the form of the GS superstring action in other type IIA backgrounds which can be obtained by reduction from locally *flat* $D = 11$ backgrounds. One particularly interesting example is the RR background representing the near-core region of a D6-brane. It can be obtained by reduction from the near-core part of the KK monopole [21] which happens to be a flat space – an ALE space with A_{N-1} singularity times a 7-dimensional Minkowski space $M^{(6,1)}$ (S^3 part of R^4 is replaced by a Hopf

[12] In the case of the KK Melvin (NS-NS) model similar procedure of starting with a covariant superstring (or membrane) action, fixing light-cone gauge and then performing the rotation in the plane gives the action (2.8) with the fermionic part $-2i\partial_a x^+ \bar\theta \Gamma^- (\delta_{ab} - \Gamma_{11}\epsilon_{ab})\mathcal{D}_b \theta$. It is easy to check explicitly that the procedures of rotation and light-cone gauge fixing indeed commute.

fibration $S^2 \times S^1$). The Green-Schwarz action for a string in the curved magnetic RR background corresponding to the near core (or decoupling [22]) region of the D6-brane can be thus obtained by the same method as used above. Starting with the membrane action in the locally flat $D = 11$ background

$$\begin{aligned} ds_{11}^2 &= -dt^2 + dx_1^2 + ... + dx_6^2 + d\rho^2 + \rho^2(d\tilde{\phi}^2 + \sin^2\tilde{\phi}\ d\tilde{\varphi}^2 + \cos^2\tilde{\phi}\ d\tilde{\psi}^2) \\ &= -dt^2 + dx_1^2 + ... + dx_6^2 + \tfrac{1}{2}Nr^{-1}dr^2 + \tfrac{1}{2}Nr(d\phi^2 + \sin^2\phi\ d\varphi^2) \\ &\quad + 2N^{-1}r[d\psi + \tfrac{1}{2}N(\cos\phi - 1)d\varphi]^2 \ , \end{aligned} \tag{5.1}$$

where $r \equiv \frac{1}{2}N^{-1}\rho^2$, $\phi = 2\tilde{\phi}$, $\varphi = \tilde{\varphi} - \tilde{\psi}$, $\psi = N\tilde{\psi} \equiv \psi + 2\pi$, and making the double dimensional reduction along $x_{11} = R_{11}\psi$ leads to the covariant Green-Schwarz action for this background. We expect that the resulting action will be simpler than (4.3) , since the basic function of the radial coordinate here is just a power of r.

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